6th China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems
June 22–25, 2010, Kyoto, Japan

Discrete optimization of modular truss network in a constrained environment

Machi Zawidzki[1], Ikuko Nishikawa[2]

[1]Ritsumeikan University, Department of Architecture and Urban Design
[2]Ritsumeikan University, Department of Human and Computer Intelligence
Kusatsu, Japan, zawidzki@gmail.com

Abstract
The paper introduces the concept of a modular spatial truss system: truss-Z. The procedure to construct and optimize the communication networks for pedestrian traffic in a given environment with truss-Z modules is shown. The truss network connects a given number of terminals and also allows the creation of closed loops and the branching of paths. The elements of the environment model real obstacles such as roads, buildings, lakes and so on. The truss must not collide either with the obstacles or with itself. Two methods of constructing the truss network in the constrained environment are presented: backtracking and alignment to a given spline. Both methods use discrete optimization to produce allowable solutions which can be globally optimized for various objectives. These include the total number of modules used (minimization), the best alignment to the given paths (minimization), the network distance (minimization) and network flow (maximization). An example is given of optimization of the network distance. A method of implementing genetic operations for global optimization of the truss network is shown.

Keywords: *modular truss system, 3D truss, discrete optimization, genetic algorithm, constraints satisfaction problem*

1. Introduction

Truss-Z is an innovative concept of a modular truss system invented by Machi Zawidzki for constructing structural communication networks for pedestrian traffic such as overpasses, ramps for visitors etc as shown in fig. 1. The same concept can be adjusted in scale and shape for different purposes such as cycle paths or other transportation tasks. Unlike available modular truss systems [1] the concept of truss-Z allows construction of complicated geometrical networks by assembling only two types of module. Usually such free-form structures require high levels of customization where many members need to be individually prefabricated or adjusted [2,3], while available systems composed of simple modules offer very limited spatial diversity [4,5].

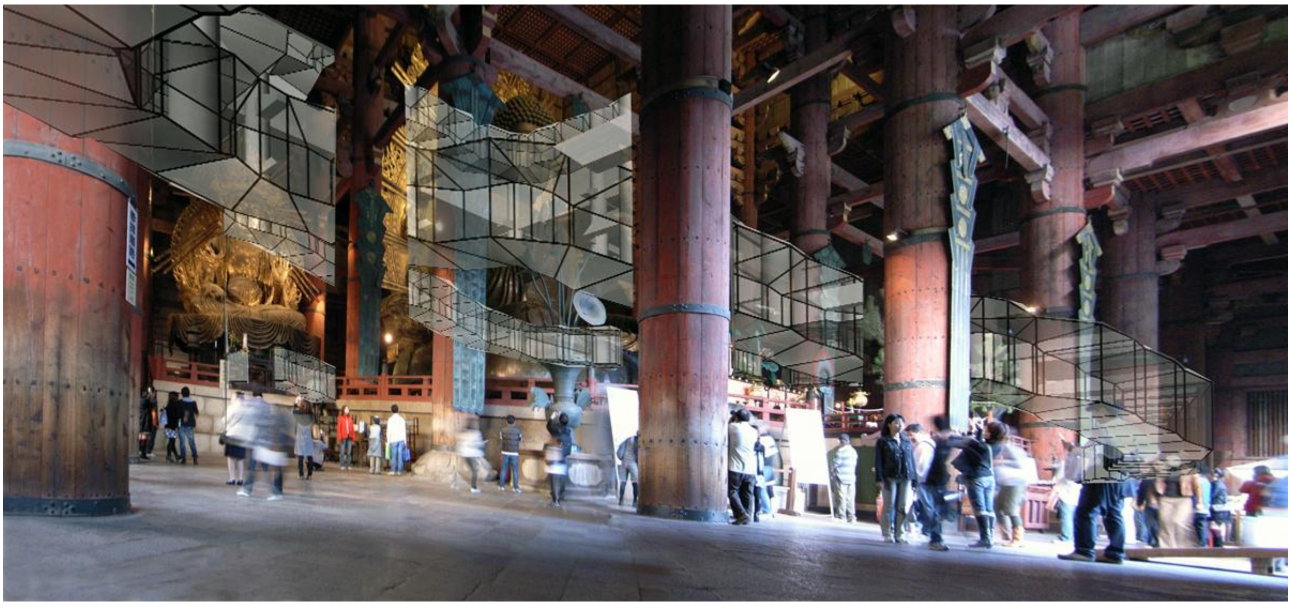


Figure 1. A conceptual example of application of the truss-Z system

- The system is flexible: realizes a large number of geometrical tasks. A single segment of the truss can link any two terminals in 3D space. The system also supports branching of segments and loops.

- The system is modular: Every segment of the truss is composed of only two modular elements. The supporting system is based on a single member. Elements are intended for prefabrication and assembly on location (preferably without the necessity for heavy equipment) or fabrication on site using templates and locally available materials (for example timber in a forest).

The truss network can be optimized according to any of the following objectives (or their combination):
- The overall number of modules used (minimization)
- The alignment to the given paths (minimization)
- The network distance (minimization)
- The flow (maximization)

The truss is modular and one network configuration differs from another by the way of connecting the units, therefore discrete optimization methods can be applied. Although the number of possible combinations of the units is finite, it can easily become extremely large, thus heuristic methods are also considered (genetic algorithm).
The support system is necessary in case of practical applications and is also under consideration but it will not be presented in this paper.

2. The module
Truss-Z is a structural spatial truss system which was originally designed for pedestrian traffic and therefore has specific geometrical properties [6].

2.1. The geometry of the module
The vertical angle of the module at the axis is 11.31° (8.05° at the long and 27.80° at the short side). The horizontal angle of the unit is 30°, therefore a right angle turn can be completed with only three units. The module supports a number of configurations including a vertical spiral. The main truss is composed of only two modules named according to the right-hand-rule: R (right) which turns left and goes up and L (left) which turns right and goes up. The L unit is a mirror reflection of the R unit. When the R unit is rotated along the vertical axis by 150° it becomes R2 (rotated right) unit and the rotation of the L unit is an L2 (rotated left) unit. All these variations have different "vectors" equivalent to the change of the position in space: "up and left" (R), "up and right" (L), "down and left" (R2) and "down and right" (L2) as shown in Figure 2. The module is a rigid spatial frame-truss hybrid.

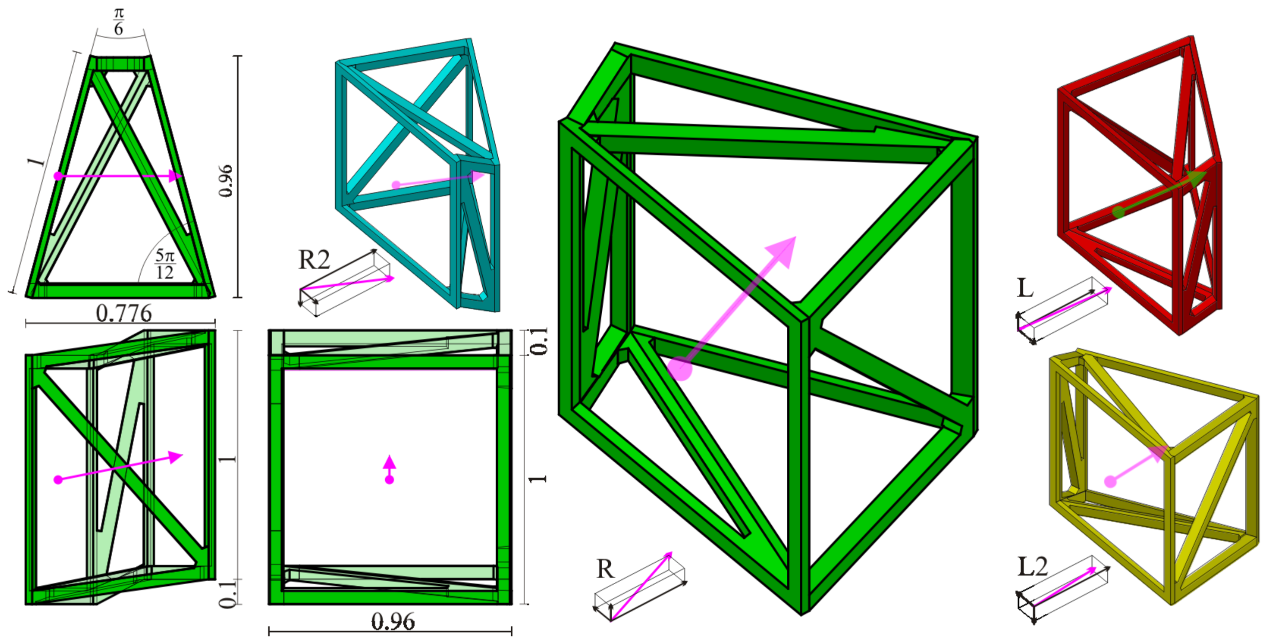


Figure 2.The geometrical properties of the module and all its variations: R, R2, L, L2.

2.2. Combining the modules
Every two units can be connected in four different ways allowing the creation of any spatial path.
Examples of sequences of the units and the resulting structures are shown in Figure 3. The coloring convention is R- green, R2- cyan, L- red, L2- yellow.

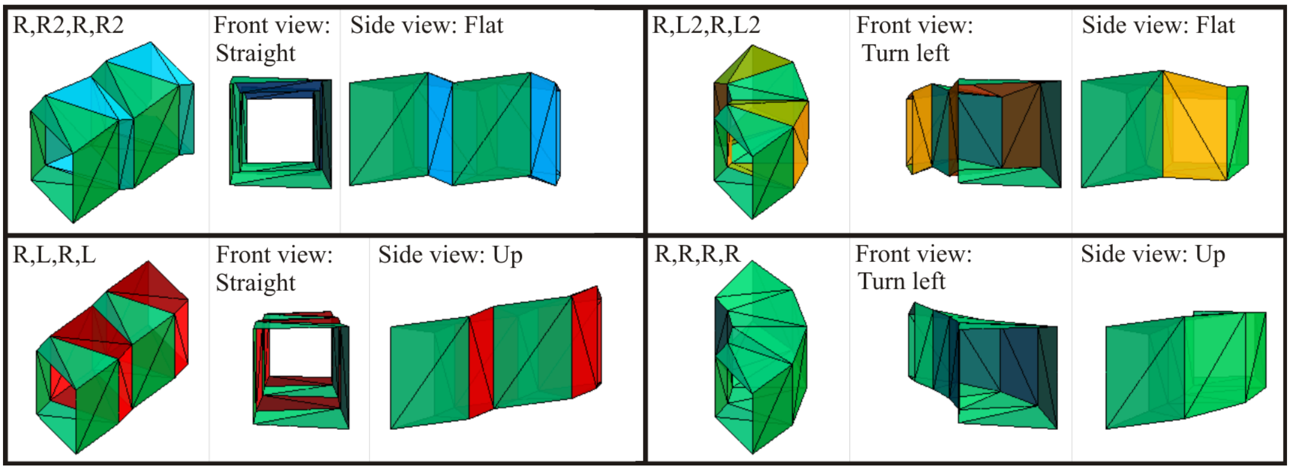


Figure 3. Examples of sequences of units and the resulting spatial structures.

2.3. The branching point

If the number of terminals is greater than two, a special way of connecting the modules (by aligning two longer sides of the units) allows the creation of a branching point as shown in figure 4.

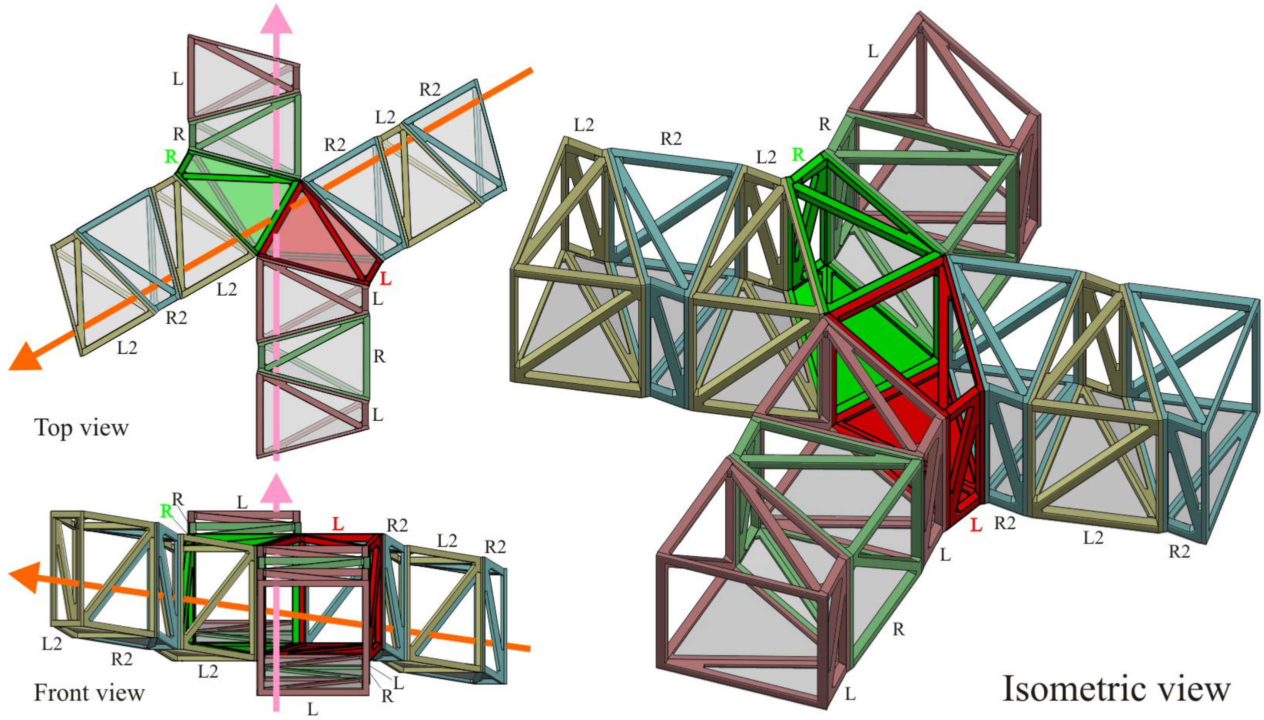


Figure 4. A junction with four branches

3. The truss network

For creating the truss network the following assumptions have been made:

1) The truss must connect any given number of terminals (points in space).
2) The segments of the truss must not collide with any objects: obstacles and other segments of the truss or the supports.

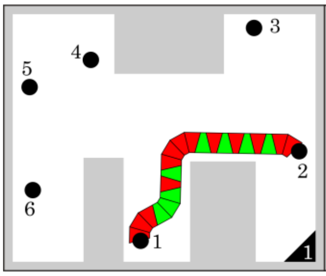
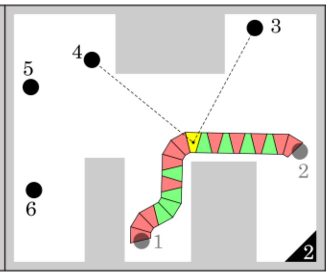
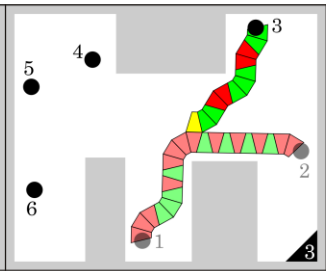
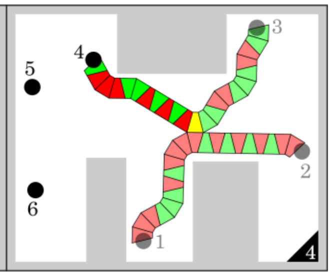
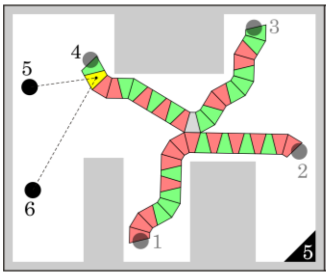
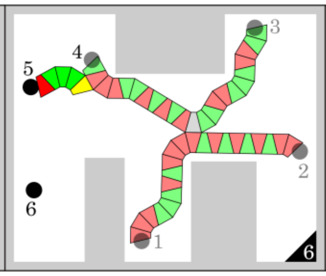
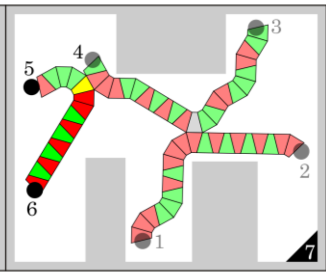
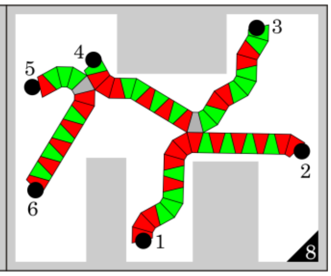
Two methods of constructing the truss network are discussed: backtrack algorithm and alignment to the given paths.

3.1. Backtracking

This procedure is a combination of Constraint Satisfaction Problem (CSP) where all the units must lie within the given area and discrete local optimization - choosing the module which lies closer to the target. In this way locally optimal solutions are found, but the completed truss network is most likely not globally optimal as it is just one of the possible configurations. The problem in general is three-dimensional, but for clarity was reduced to the 2D plane, therefore instead of four possible variations of a unit there are only two: R (right) and L (left). An example of the procedure to create a truss network for six terminals in an constrained environment is shown in table 1. The truss modules can be placed within the white area only. The first (starting) terminal is chosen arbitrarily and the first segment of the truss connecting the first pair of consecutive terminals is constructed according to the following algorithm:

1) At each step a module (R or L) whose centroid is closer (minimization) to the target (second terminal) is chosen.
2) If any point of the truss module lies outside the allowed area (Constraints Satisfaction), the procedure steps back and switches the last unit (R→L or L→R), and continues as in 1),
3) If any point of the truss module still lies outside the allowed area the procedures steps farther back, switches the units and continues as in 2) until none of the modules lie outside the allowed area.

Table 1. The truss network procedure for an example with six terminals at constrained environment

			
1) Make the first branch from the starting point to the next consecutive terminal.	2) Choose an available segment from the first segment so that the sum of the distances to the next pair of terminals is minimal.	3) Construct a new branch to the next given terminal.	4) Construct the next branch in the opposite direction (if applicable).
			
5) As in 2) choose the new branching point in the previous branch.	6) As in 3) construct a new branch to the next given terminal.	7) As in 4) construct the next branch in the opposite direction (if applicable).	8) Completed truss network with 59 modules.

In order to construct the next segment of the truss, a branching unit is selected from the first segment. The branching can be executed by attaching an extra module to a longer side of a unit in the first segment. Therefore not all the segments are available for serving as a branching point-CSP (in the table 2.2 only L (left) modules indicated by red can be used for branching towards terminals 3 and 4). From the available segments, one is chosen so that the sum of the distances to the next terminals is minimal (minimization).

3.2. Alignment to the given path

In this method the truss segments are aligned to given 3D splines. At each step one of four variations of units (two modules and two rotations) which gives the minimum value of the objective function (as shown in figure 5) is chosen:

$$\text{Minimize}[u\, d/b+(1-u)(1-|\boldsymbol{v}\bullet r'[\text{p}]|)], \text{ where}$$

d is the smallest distance between the centroid (C) of a unit and the curve in point p;
$\boldsymbol{v}$ is the vector of the unit;
r'[p] is the direction of the spline in point p;
u and b are parameters described below.

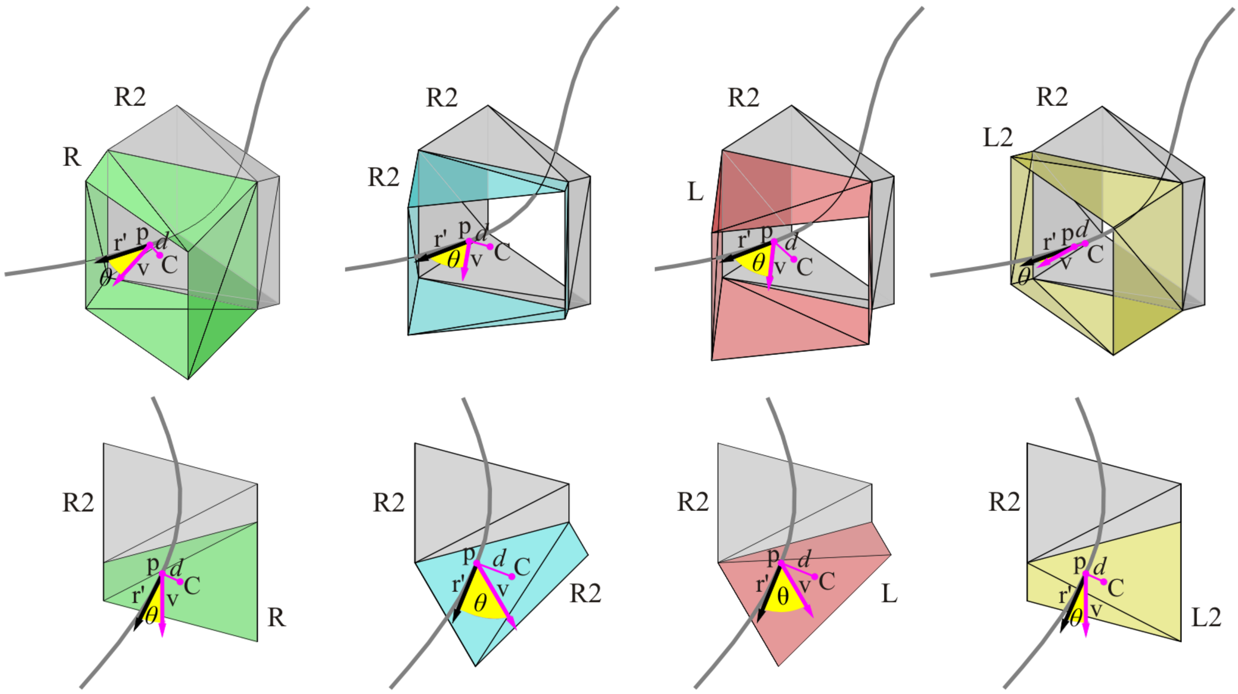


Figure 5. Four possible configurations for the next unit. Top row: isometric views, bottom row: top views.

The parameter *u* is a weight ranging from 0 to 1 and moves the influence from the normalized dot product of the direction of the curve and the vector of the unit to the distance between the centroid of a unit and the curve. Since the objective function depends both on *distance* and *angle* which cannot be normalized, the parameter *b* adjusts the ratio between them. The values of parameters *u* and *b* were optimized so the number of units to follow a given path was minimal as shown in figure 6.

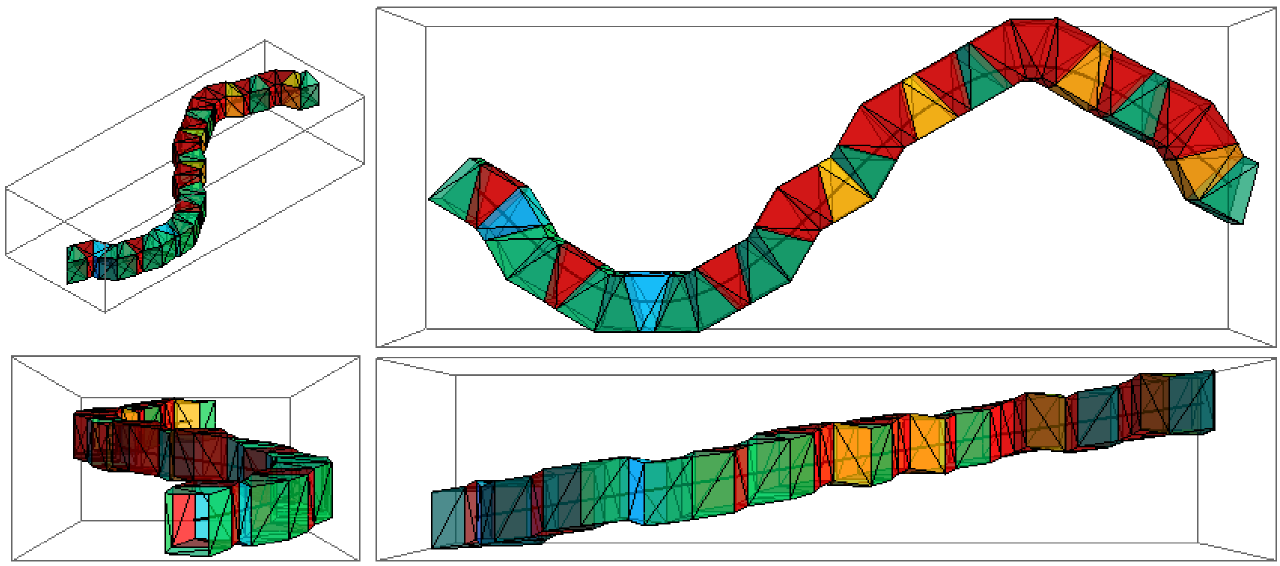


Figure 6. u = 0.5, b = 100; The balance between maintaining a small distance to the guide path and following its curvature. The number of units to follow the given path is 33 (minimum). The list of units: {R,L,R2,R,R, L,R,R,R2,R,R,L,R,R,L,L,L2,R,L,L,L2,L,R,L,L,L,L2,L,R,L,L,L2,R}.

The algorithm operates on a single path as well as a list of sub-paths. The procedure for constructing a truss network with multiple segments consists of the following steps:

1) At first all the segments of the truss containing two junction points are constructed along the given spline as shown in figure 7.1.
2) The junction units are completed by attaching the “mirrored” units (Figure 7.2).
3) These units become starting points for the next segments of the truss. (Figure 7.2)

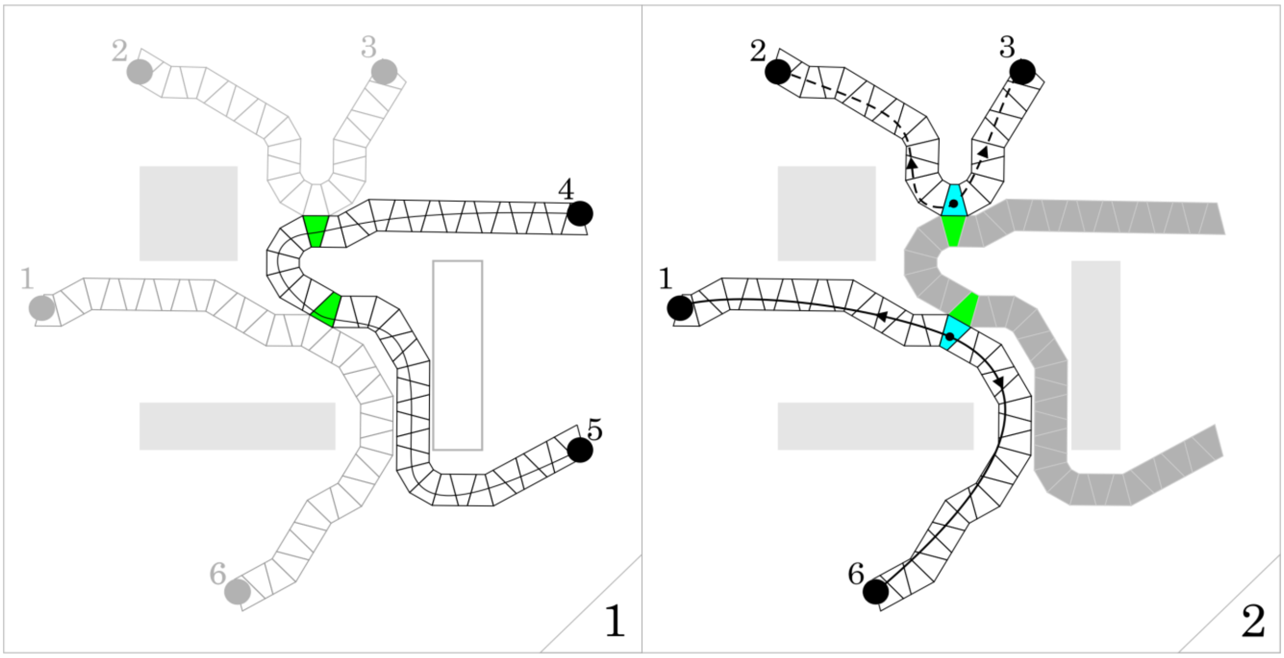


Figure 7. The sequence of constructing the segments of the truss network.

4. Global optimization

The configurations presented above are possible allowable solutions for given terminals and constraints. Most often they are not globally optimal and can be further optimized for various objective functions, for example the minimal network distance. The network distance is the sum of all traverse lengths connecting every pair of terminals. An example of two networks with different network distances is shown in table 2.

4.1. Genetic algorithm

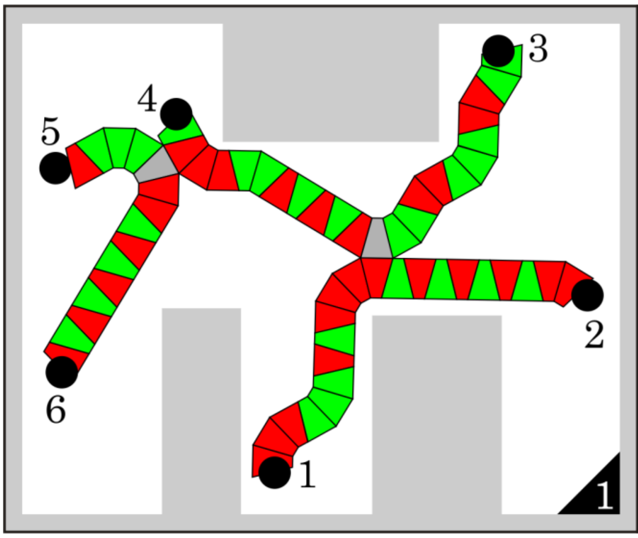
Since the truss system is modular, the space of all possible solutions is always finite therefore the global optimization can be done by an intensive search. Nevertheless, for a larger number of units heuristic methods should be applied - for example a genetic algorithm.

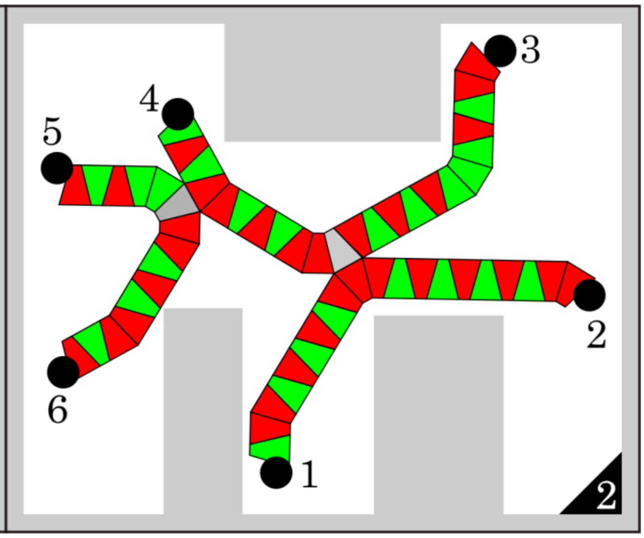
4.2. The truss genotype

The truss network can be encoded in a genotype as shown in figure 8. Such code can be uniquely decoded into a phenotype and evaluated against structural requirements - for example whether the truss overlaps with any objects or intersects with itself or against evaluation function - for example the distances of the leaf-units (the last units in the truss segments) to the given terminals.

Table 2. Two truss networks with the same number of units (59) but different network distances (335 and 318)

The truss network produced by the backtrack algorithm	More efficient network (minimized network distance)





Pair of terminals	Distance		Pair of terminals	Distance	
1-2	21	Total **335**	1-2	20	Total <u>**318**</u>
1-3	23		1-3	22	
1-4	23		1-4	20	
1-5	26		1-5	22	
1-6	32		1-6	26	
2-3	21		2-3	23	
2-4	21		2-4	21	
2-5	24		2-5	23	
2-6	30		2-6	27	
3-4	23		3-4	23	
3-5	26		3-5	25	
3-6	32		3-6	29	
4-5	6		4-5	9	
4-6	12		4-6	13	
5-6	15		5-6	15	

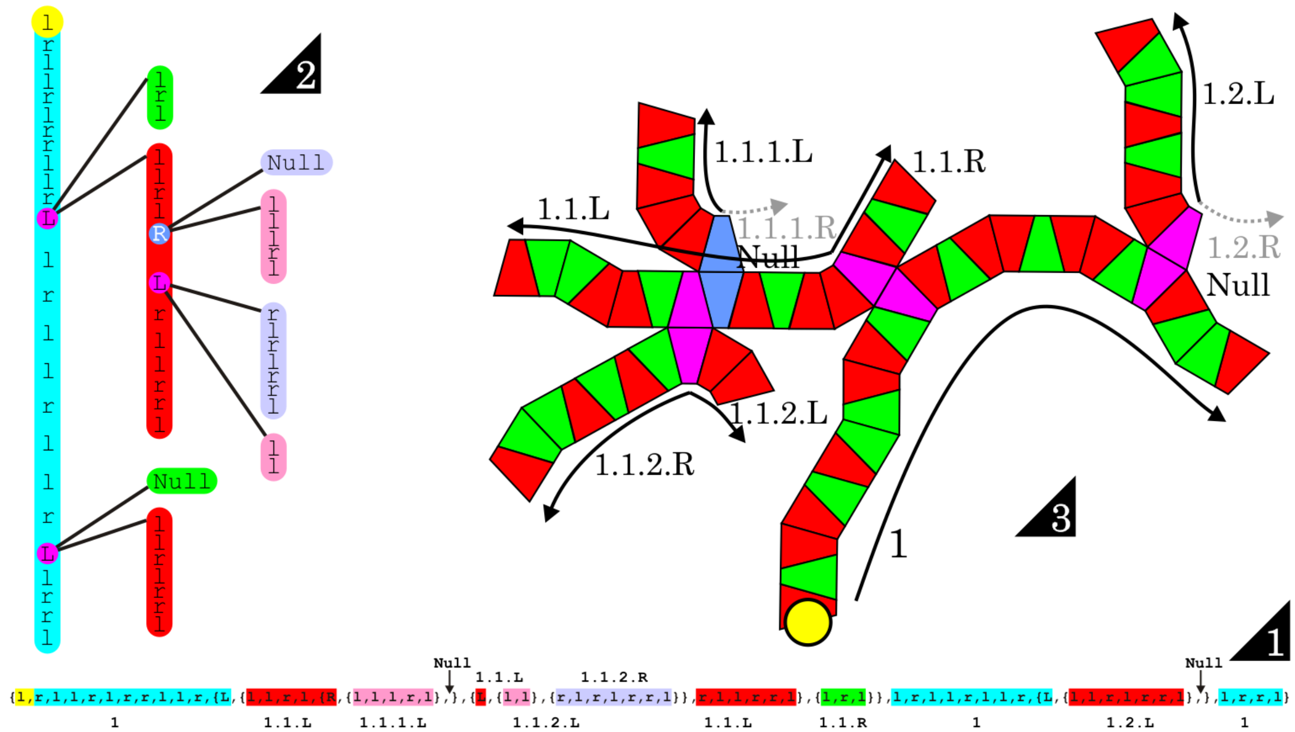


Figure 8. An example of encoding of a truss network: 1) a genotype, 2) graphic visualization of a genotype, 3) a phenotype. Arrows in figure 3 explain the naming convention for the units, which is relative to the branching point.

4.3. Genetic operations

The genotype of the truss can be subject to genetic operations: mutation and crossover as shown in figure 9.

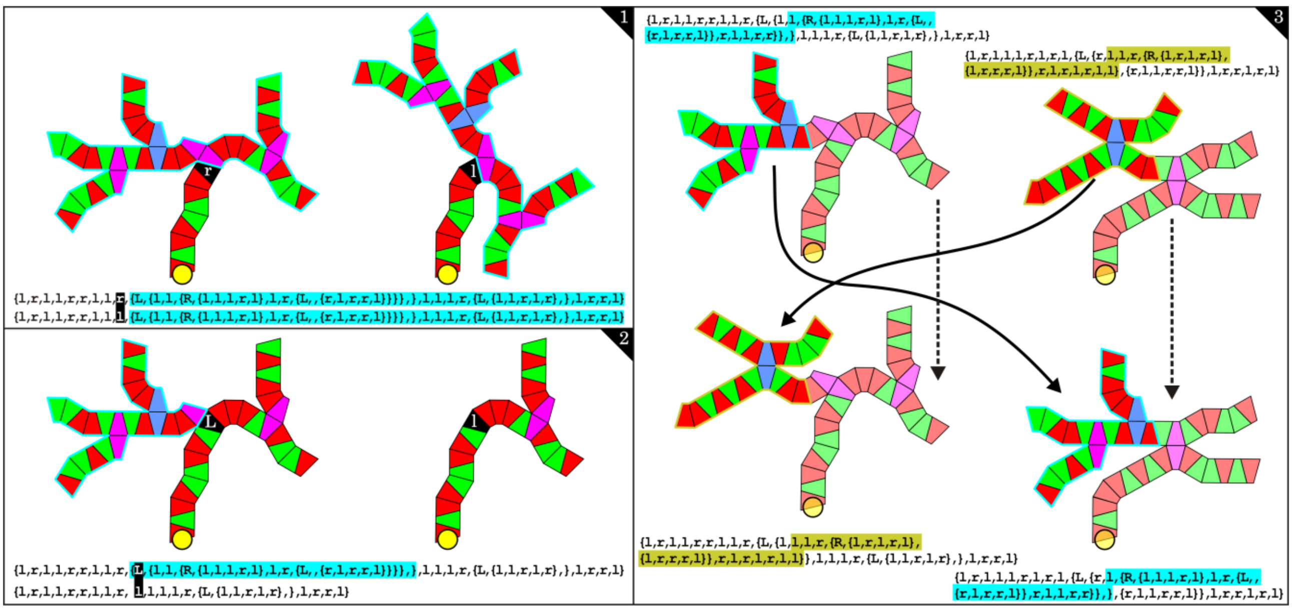


Figure 9. Examples of genetic operations and resulting phenotypes. 1) Mutation of a gene: r (regular right module) → l (regular left module): part of the truss is reoriented. 2) L → l (a branching left module becomes a regular left module: a whole branch disappears). 3) A crossover.

5. Summary and Future Work

1) An example of optimizing the layout of a truss network with truss-Z system for pedestrian traffic at given conditions has been demonstrated.
2) The system can be locally modified (without influencing the remainder of the structure) and adapted to changing conditions, for example by redirecting or increasing the capacity of pedestrian traffic according to the time of the day [7].
3) Development of a joint system for on-site assembly is under consideration.
4) The global optimization of the truss network including the supports is under consideration.

References

1. Staib G., Dörrhöfer A., Rosenthal M., Components and Systems: Modular Construction Design, Structure, New Technologies, Birkhäuser, 2008.
2. Spuybroek L., NOX, Thames & Hudson, pp. 142-157, 2004.
3. Spuybroek L., The architecture of variation, Thames & Hudson, pp. 127-129, 2009.
4. "Allround Modular Truss System" by Wilhelm Layher GmbH & Co. KG http://www.layher.co.nz/scaffold-project-allround-modular-truss-system.html
5. Modular truss systems: (US Patents 5964068, 5826384, 7347030, 5390463)
6. Bowman R., An introductory guide to the slip resistance of pedestrian surface materials, Strathfield, NSW: Standards Australia; Wellington: Standards New Zealand, 1999.
7. Luchsinger R.H., Crettol R., Adaptable Tensairity, IASS International Conference on Adaptability in Design and Construction, Eindhoven, 2006.